APPENDIX IV

MAGNETOELASTIC ENERGY ABOUT A SPHERICAL PORE

In this appendix, the magnetoelastic energy density about a spherical pore in an isotropic elastic magnetic medium subject to hydrostatic pressure will be derived. Figure IV.1 should be referred to for symbols.

It is first necessary to find the strain field about a pore subject to a limiting boundary condition of hydrostatic strain. This is accomplished by finding the displacement field. The displacement field, because of symmetry, is of the form

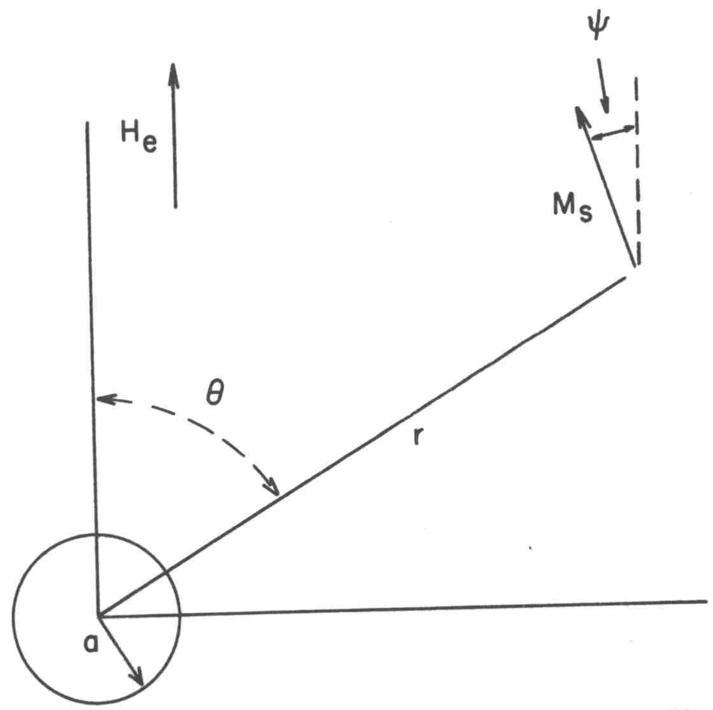

Fig. IV.1.--Spherical cavity of radius a. $M_s \cos\psi$ is the magnetization in the direction of the applied field at the spherical coordinate (r,θ,ϕ). There is azimuthal symmetry.

$$\vec{u} = u_r(r)\hat{i}_r.$$

The radial displacement must satisfy Laplace's equation. In spherical coordinates, this becomes

$$\frac{d}{dr}\left[r^2\left(\frac{du_r}{dr}\right)\right] - 2u_r = 0.$$

This has the solution

$$u_r = Ar + \frac{B}{r^2}.$$

The limiting boundary condition $(r \to \infty)$ is that the strain be hydrostatic.

$$\left.\frac{du_r}{dr}\right|_{r=\infty} = \left.e_{rr}\right|_{r=\infty} = -\frac{K_T P}{3},$$

where K_T is the isothermal compressibility. The boundary condition at $r = a$ is that the normal stress be zero.

$$\sigma_{rr} = \frac{E}{(1+\nu)(1-2\nu)}((1-\nu)e_{rr} + \nu(e_{\theta\theta} + e_{\phi\phi})) = 0$$

where E is Young's modulus and ν is Poisson's ratio. This boundary condition becomes

$$\left.\frac{du_r}{dr}\right|_{r=a} = \left.e_{rr}\right|_{r=a} = \frac{\nu}{(1-2\nu)} K_T P.$$

The radial displacement field satisfying the boundary conditions is